Reducibility and thermal scaling in percolation

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Cluster distributions of a simple cubic lattice of side six were examined as a function of bond breaking probability, p_{break}. Evidence of reducibility and thermal scaling was found, suggesting that they are fundamental features rather than epiphenomena of complex systems.

Reducibility indicates that for each bin in p_{break} the cluster multiplicities, N, are distributed according to a binomial or Poissonian law. Their multiplicity distributions, P_N, can be *reduced* to a *one-cluster production probability* p, according to the binomial or Poissonian law:

$$\begin{aligned} P_N^M &= \frac{M!}{M!(M-N)!} p^N (1-p)^{M-N}; \\ P_N &= e^{-\langle N \rangle} \frac{1}{N!} \langle N \rangle^N, \end{aligned} \tag{1}$$

where M is the total number of trials.

The ratio of the variance to the mean, $\sigma_A^2 / \langle N_A \rangle$, of the multiplicity distribution for each cluster of size A is an indicator of the nature of the distribution. The observed ratio is near one (Poissonian limit) for all p_{break}. See Fig. 1.

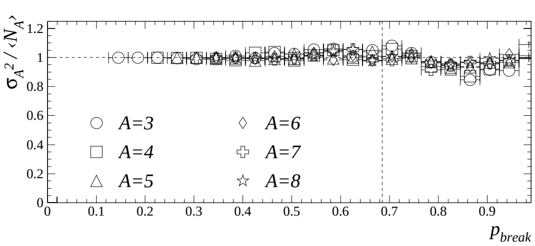

Figure 1: Ratio of the variance to the mean number of clusters of size A versus p_{break}; location of the critical point shown by vertical dashed line.

Thermal scaling refers to the feature that p behaves with temperature T as a Boltzmann factor: $p \propto \exp(-B/T)$. A plot of $\ln p$ vs. $1/T$ (Arrhenius plot) will be linear if p is a Boltzmann factor with B as the one-cluster production barrier.

Thermal scaling was observed as a Boltzmann factor when $\ln \langle n_A \rangle$ was plotted as a function of

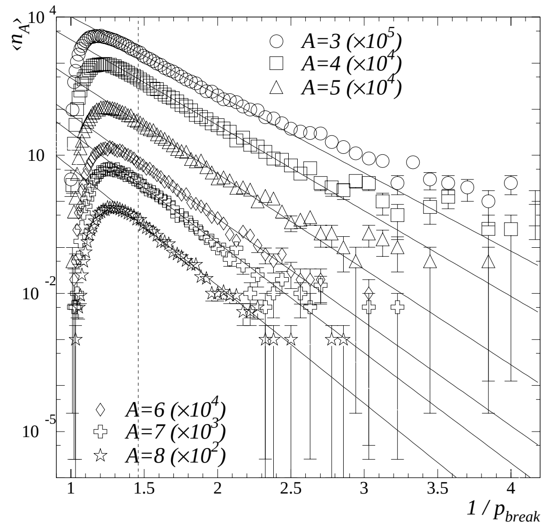

Figure 2: Normalized average cluster multiplicity versus $1/p_{break}$ for clusters of size A. Solid lines show Arrhenius fits.

$1/p_{break}$; here the common practice of replacing T with p_{break} was followed. See Fig. 2. Arrhenius plots for individual clusters of size A are linear over several orders of magnitude.

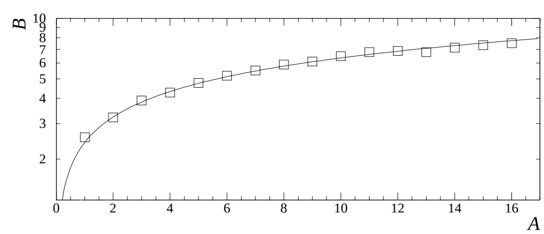

Figure 3: The power law relationship between the Arrhenius barrier, B, and the cluster size A.

Interpreting the Boltzmann factor in the terms of the Fisher Droplet Model yields a power law relating B to the size of a cluster: $B = c_0 A^{\sigma}$. Fitting the extracted barriers B as a function of A gave an exponent equal to 0.42 ± 0.02 in agreement with $\sigma = 0.45$ for 3D percolation. See Fig. 3.